CHAPTER - 2

Differential Eq^n of First Order and First degree

Def^n:- A general differential eq^n of First order and First degree is an equation of the form $\frac{dy}{dx} = f(x,y)$ Or $Mdx + Ndy = 0$; M and N may be both functions of x and y.

Variable separable Form :-

The general form of a differential equation of Ist order and Ist degree is

$$\frac{dy}{dx} = F(x,y) \quad \text{—(1)} \quad ; \quad F(x,y) = (g(x), h(y))$$

Then the differential equation (1) on separating the variables becomes

$$\frac{dy}{h(y)} = g(x)\, dx \quad \text{—(2)}$$

Integrating (2), we get

$$\int \frac{dy}{h(y)} = \int g(x)\, dx + C \; ; \; C \text{ is an arbitrary constant}$$

This represents the general sol^n of differential equation (1)

OR

$$\frac{dy}{dx} = XY \quad \text{—(3)} \quad ; \; X \text{ is function of } x \text{ only}, \; Y \text{ is function of } y \text{ only.}$$

$$\frac{dy}{Y} = X\, dx \quad \text{—(4)}$$

$$\Rightarrow \int \frac{dy}{Y} = \int X\, dx + C \; ; \; C \text{ is an arbitrary constant}$$

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Some important numerical on above topic.

1. Solve the differential eqn.

(a) $\frac{dy}{dx} + y = 1$

Solⁿ:- we have $\frac{dy}{dx} + y = 1$ — (1)

eqⁿ (1) is of the form $y' = F(x, y)$
OR
$y' = XY$

(1) ⇒ $\frac{dy}{dx} = 1 - y$ ⇒ $\frac{dy}{1-y} = dx$ — (2)

Integrating eqⁿ (2), we get

$$\int \frac{dy}{1-y} = \int dx + c$$

$$\Rightarrow \frac{\log(1-y)}{-1} = x + c$$

$$\Rightarrow \log(1-y) = -(x+c)$$

$$\Rightarrow 1 - y = e^{-(x+c)}$$

$$\Rightarrow 1 - y = e^{-x} e^{-c} = Ae^{-x} \quad ; \; A = e^{-c}$$

$$\Rightarrow y = 1 + Ae^{-x}$$

which is the required solⁿ

(2) $y \log y \, dx - x \, dy = 0$

Sol^n:- We have $y \log y \, dx - x \, dy = 0$ —— (1)

$\Rightarrow y \log y \, dx = x \, dy$

$\Rightarrow x \, dy = y \log y \, dx$

$\Rightarrow \dfrac{dy}{y \log y} = \dfrac{dx}{x}$ —— (2)

Integrating (2), we get

$\displaystyle\int \frac{dy}{y \log y} = \int \frac{dx}{x} + C$

Put $\log y = t$

$\Rightarrow \frac{1}{y} dy = dt$

$\Rightarrow \displaystyle\int \frac{dt}{t} = \int \frac{dx}{x} + C \Rightarrow \log|t| = \log|x| + \log|c|$

$\Rightarrow \log|\log y| = \log(xc) \Rightarrow \log y = xc$

which is the required sol^n.

3. $(e^x + e^{-x}) \dfrac{dy}{dx} = e^x - e^{-x}$

Sol^n:- We have $(e^x + e^{-x}) \dfrac{dy}{dx} = e^x - e^{-x}$ —— (1)

$\Rightarrow \dfrac{dy}{dx} = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

$\Rightarrow dy = \dfrac{e^x - e^{-x}}{e^x + e^{-x}} dx$ —— (2)

Integrating ②

$$\int dy = \int \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}}\, dx + c$$

put $e^{x} + e^{-x} = t \Rightarrow (e^{x} - e^{-x})\, dx = dt$

$$\Rightarrow \int dy = \int \frac{dt}{t} + c$$

$$\Rightarrow y = \log|t| + c$$

$\Rightarrow y = \log|e^{x} + e^{-x}| + c$ which is the required sol^{n}.

4. $\tan y \frac{dy}{dx} = \sin(x+y) + \sin(x-y)$ — ①

$$\Rightarrow \tan y \frac{dy}{dx} = 2\sin x \cos y$$

$$\Rightarrow \frac{\tan y\, dy}{\cos y} = 2\sin x\, dx. \quad \text{— ②}$$

Integrating ②.

$$\int \sec y \tan y\, dy = 2\int \sin x\, dx + c$$

$$\Rightarrow \sec y = -2\cos x + c$$

which is the required sol^{n}

5. solve the differential eq^{n}

$x(1+y^2)\, dx + y(1+x^2)\, dy = 0$

Sol^{n}:- we have $x(1+y^2)\, dx + y(1+x^2)\, dy = 0$ — ①

$\Rightarrow y(1+x^2)dy = -x(1+y^2)dx$

$\Rightarrow \dfrac{y}{1+y^2}dy = \dfrac{-x}{1+x^2}dx$ —②

Integrating. $\displaystyle\int \frac{2y}{1+y^2}dy = -\int \frac{2x}{1+x^2}dx + c$

put $1+y^2 = t$, $1+x^2 = z$

$2y\,dy = dt$, $2x\,dx = dz$

$\Rightarrow \displaystyle\int \frac{dt}{t} = -\int \frac{dz}{z} + c$

$\Rightarrow \log|t| = -\log|z| + \log c$

$\Rightarrow \log|t| + \log|z| = \log c$

$\Rightarrow \log(tz) = \log c$

$\Rightarrow tz = c$

$\Rightarrow (1+y^2)(1+x^2) = c$ which is the required soln.

6. Solve the D.E. :-

$e^x \tan y\,dx + (1-e^x)\sec^2 y\,dy = 0$

soln:- we have $e^x \tan y\,dx + (1-e^x)\sec^2 y\,dy = 0$ —①

$\Rightarrow (e^x-1)\sec^2 y\,dy = e^x \tan y\,dx$

$\Rightarrow \dfrac{\sec^2 y}{\tan y}dy = \dfrac{e^x}{e^x-1}dx$

Integrating $\displaystyle\int \frac{\sec^2 y}{\tan y}dy = \int \frac{e^x}{e^x-1}dx + c$

Put $\tan y = t_1$, $e^x - 1 = t_2$

$\sec^2 y\, dy = dt_1$, $e^x dx = dt_2$

$\Rightarrow \int \frac{dt_1}{t_1} = \int \frac{dt_2}{t_2} + C$

$\Rightarrow \log|t_1| = \log|t_2| + \log C$

$\Rightarrow t_1 = t_2 C$

$\Rightarrow \tan y = (e^x - 1) C$. which is the required sol^n

7. Find the equation to the curve represented by $(y - xy)dx + (x + xy)dy = 0$ and passing through the point $(1,1)$

sol^n:- We have

$(y - xy)dx + (x + xy)dy = 0$

$(1,1)$
$x = 1$
$y = 1$

$\Rightarrow y(1-x)dx + x(1+y)dy = 0$

$\Rightarrow x(1+y)dy = y(x-1)dx$

$\Rightarrow \frac{1+y}{y} dy = \frac{x-1}{x} dx$

$\Rightarrow \int \left(\frac{1}{y} + 1\right) dy = \int \left(1 - \frac{1}{x}\right) dx + C$

$\Rightarrow \log y + y = x - \log x + C$

$\Rightarrow \log y + \log x - x + y = C$ —— ①

passing through $(1,1)$

putting $x = 1$, $y = 1$ in eq^n ①